Flow Routing Methods

Abhinav Shukla
Scientist/Engineer ‘SD’
Disaster Management Support Group

What is FLOW ROUTING?

✓ “Flow routing is a technique of determining the flow hydrograph at a section of a river by utilizing the data of flood flow at one or more upstream sections.”

( Subramanya, 1984)

![Diagram showing flow routing process]
Types of flow routing

Lumped/hydrologic:
- Flow \( \rightarrow f(\text{time}) \)
- Continuity equation and Flow/Storage relationship

Distributed/hydraulic:
- Flow \( \rightarrow f(\text{space, time}) \)
- Continuity and Momentum equations

Hydrologic flow routing
Channel Routing

The total volume in storage for a channel reach having a flood wave can be considered as prism storage + wedge storage.

Prism storage: The volume that would exist if uniform flow occurred at the downstream depth i.e. the volume formed by an imaginary plane parallel to the channel bottom drawn at the outflow section water surface.

Wedge storage: It is the wedge like volume formed between the actual water surface profile and the top surface of the prism storage.
Hydrologic river routing (Muskingum Method)

Wedge storage in reach

\[ S_{\text{Prism}} = KQ \]
\[ S_{\text{Wedge}} = KX(I - Q) \]

- \( K \) = travel time of peak through the reach
- \( X \) = weight on inflow versus outflow \((0 \leq X \leq 0.5)\)
- \( X = 0 \rightarrow \text{Reservoir, storage depends on outflow, no wedge} \)
- \( X = 0.0 \rightarrow 0.3 \rightarrow \text{Natural stream} \)

\[ S = KQ + KX(I - Q) \]
\[ S = K[XI + (1 - X)Q] \]

Muskingum Method (Cont.)

\[ S = K[XI + (1 - X)Q] \]
\[ S_{j+1} - S_j = K\{[XI_{j+1} + (1 - X)Q_{j+1}] - [XI_j + (1 - X)Q_j] \} \]

Recall:

\[ S_{j+1} - S_j = \frac{I_{j+1} + I_j}{2} \Delta t - \frac{Q_{j+1} + Q_j}{2} \Delta t \]

Combine:

\[ Q_{j+1} = C_1 I_{j+1} + C_2 I_j + C_3 Q_j \]

\[ C_1 = \frac{\Delta t - 2KX}{2K(1 - X) + \Delta t} \]
\[ C_2 = \frac{\Delta t + 2KX}{2K(1 - X) + \Delta t} \]
\[ C_3 = \frac{2K(1 - X) - \Delta t}{2K(1 - X) + \Delta t} \]

If \( I(t) \), \( K \) and \( X \) are known, \( Q(t) \) can be calculated using above equations
Muskingum - Example

- **Given:**
  - Inflow hydrograph
  - $K = 2.3$ hr, $X = 0.15$, $\Delta t = 1$ hour,
    Initial $Q = 85$ cfs
- **Find:**
  - Outflow hydrograph using Muskingum routing method

\[
C_1 = \frac{\Delta t - 2KX}{2K(1-X)+\Delta t} = \frac{1 - 2 \times 2.3 \times 0.15}{2 \times 2.3(1 - 0.15) + 1} = 0.0631
\]
\[
C_2 = \frac{\Delta t + 2KX}{2K(1-X)+\Delta t} = \frac{1 + 2 \times 2.3 \times 0.15}{2 \times 2.3(1 - 0.15) + 1} = 0.3442
\]
\[
C_3 = \frac{2K(1-X) - \Delta t}{2K(1-X)+\Delta t} = \frac{2 \times 2.3 \times (1 - 0.15) - 1}{2 \times 2.3(1 - 0.15) + 1} = 0.5927
\]

Muskingum – Example (Cont.)

\[Q_{j+1} = C_1I_{j+1} + C_2I_j + C_3Q_j\]

$C_1 = 0.0631$, $C_2 = 0.3442$, $C_3 = 0.5927$
**Example 8.5** Route the following flood hydrograph through a river reach for which $K = 12.0$ h and $x = 0.20$. At the start of the inflow flood, the outflow discharge is $10$ m$^3$/s.

<table>
<thead>
<tr>
<th>Time (h)</th>
<th>Inflow (m$^3$/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>10</td>
</tr>
<tr>
<td>6</td>
<td>20</td>
</tr>
<tr>
<td>12</td>
<td>50</td>
</tr>
<tr>
<td>18</td>
<td>60</td>
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<tr>
<td>24</td>
<td>55</td>
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<tr>
<td>30</td>
<td>45</td>
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<tr>
<td>36</td>
<td>35</td>
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<tr>
<td>42</td>
<td>27</td>
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<tr>
<td>48</td>
<td>20</td>
</tr>
<tr>
<td>54</td>
<td>15</td>
</tr>
</tbody>
</table>

**Solution:** Since $K = 12$ h and $2Kx = 2 \times 12 \times 0.2 = 4.8$ h, $\Delta t$ should be such that $12 > \Delta t > 4.8$. In the present case $\Delta t = 6$ h is selected to suit the given inflow hydrograph ordinate interval.

Using Eqs. (8.16-a, b & c) the coefficients $C_0$, $C_1$, and $C_2$ are calculated as

$$C_0 = \frac{-12 \times 0.20 + 0.5 \times 6}{12 - 12 \times 0.2 + 0.5 \times 6} = \frac{0.6}{12.6} = 0.048$$

$$C_1 = \frac{12 \times 0.2 + 0.5 \times 6}{12.6} = 0.429$$

$$C_2 = \frac{12 - 12 \times 0.2 - 0.5 \times 6}{12.6} = 0.523$$

For the first time interval, 0 to 6 h,

$I_1 = 10.0$  \quad $C_0 I_1 = 4.29$

$I_2 = 20.0$  \quad $C_0 I_2 = 9.6$

$Q_1 = 10.0$  \quad $C_0 Q_1 = 5.23$

From Eq. (8.16) \quad $Q_2 = C_2 I_2 + C_1 I_1 + C_0 Q_1 = 10.48$ m$^3$/s

For the next time step, 6 to 12 h, $Q_2 = 10.48$ m$^3$/s. The procedure is repeated for the entire duration of the inflow hydrograph. The computations are done in a tabular form as shown in Table 8.4. By plotting the inflow and outflow hydrographs the attenuation and peak lag are found to be 10 m$^3$/s and 12 h respectively.

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**Table 8.4** Muskingum Method of Routing—Example 8.5

<table>
<thead>
<tr>
<th>$\Delta t$ = 6 h</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time (h)</td>
</tr>
<tr>
<td>----------</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>6</td>
</tr>
<tr>
<td>12</td>
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<td>54</td>
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</tbody>
</table>
Hydrologic flow routing
Modified Pul’s Method

\[ I - Q = \frac{dS}{dt} \]

\[ \bar{I} \Delta t - \bar{Q} \Delta t = \Delta S \]

\[ \left( \frac{l_1 + l_2}{2} \right) \Delta t - \left( \frac{Q_1 + Q_2}{2} \right) \Delta t = S_2 - S_1 \]

\[ \left( \frac{l_1 + l_2}{2} \right) \Delta t - \left( S_1 - \frac{Q_1 \Delta t}{2} \right) = \left( S_2 + \frac{Q_2 \Delta t}{2} \right) \]

Flow routing in channels

- Distributed Routing
- St. Venant equations
  - Continuity equation
    \[ \frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = 0 \]
  - Momentum Equation
    \[ \frac{1}{A} \frac{\partial Q}{\partial t} + \frac{1}{A} \frac{\partial}{\partial x} \left( \frac{Q^2}{A} \right) + g \frac{\partial y}{\partial x} - g(S_o - S_f) = 0 \]

What are all these terms, and where are they coming from?
Continuity Equation

Elevation View

Plan View

Momentum Equation

- From Newton’s 2\textsuperscript{nd} Law:
- Net force = time rate of change of momentum
Forces acting on the C.V.

- \( F_g \) = Gravity force due to weight of water in the C.V.
- \( F_f \) = Friction force due to shear stress along the bottom and sides of the C.V.
- \( F_e \) = Contraction/expansion force due to abrupt changes in the channel cross-section
- \( F_w \) = Wind shear force due to frictional resistance of wind at the water surface
- \( F_p \) = Unbalanced pressure forces due to hydrostatic forces on the left and right hand side of the C.V. and pressure force exerted by banks

Momentum Equation

\[
\sum F = \frac{d}{dt} \iiint_{c.v.} V \rho \, dV + \iint_{c.s.} V \rho V \, dA
\]

Sum of forces on the C.V.  
Momentum stored within the C.V  
Momentum flow across the C.S.

\[
\frac{1}{A} \frac{\partial Q}{\partial t} + \frac{1}{A} \frac{\partial}{\partial x} \left( \frac{Q^2}{A} \right) + g \frac{\partial v}{\partial x} - g(S_o - S_f) = 0
\]
Flood Modeling Processes that can be simulated

- Evaporation
- Surface runoff
- Precipitation
- Coastal

Hydrological Model

Hydrodynamic Model

Groundwater

Kinematic Wave
Diffusion Wave
Dynamic Wave
Hydrological Model Classification

Lumped
Parameters assigned to each sub-basin

Semi-Distributed
Parameters assigned to each grid cell, but cells with same parameters are grouped

Fully-Distributed
Parameters assigned to each grid cell

BROAD METHODOLOGY

Stages in the Flood Forecasting
- Computing runoff volume
  - SCS Curve Number Loss
- Modelling direct runoff
  - SCS Transform Method
- Flood Routing
  - Muskingum
- Calibration of the model
- Model validation

Spatial and non-spatial Database

Static Data Used
- Land use, Soil Texture, CARTO DEM

Dynamic Data
- Historic Hydro-meteorological Data
- Real-time 3 hr. hydrological data (CWC)
- Real time In-situ 24 hr Rainfall Data (IMD)
- 24 hr Rainfall Forecast Data at 3 hr frequency (9 km grids from IMD)
- Monthly ET data (computed)
**HEC HMS (Curve Number Method)**

GIS Preprocessing (DEM, Soil, Land Use) → Preprocessed GIS Database → HEC-GeoHMS → HEC-HMS Inputs → Calibration and Validation → Routing Flood Hydrograph → Flood Characteristic (Peak Discharge and Volume)

**SCS Curve Number**

\[ P_{IN} = \frac{R_{IN}}{S} \]

\[ S = \frac{254}{C_{NO}} - \frac{1000}{C_{NI}} \]

slope adjusted curve number

\[ C_{NO} = \frac{1}{2} \left( C_{NO} - C_{VI} \right) \left( 1 - 2e^{-0.85 \frac{R_{IN}}{C_{NO}}} \right) + C_{NI} \]

**MIKE 11 (NAM (Nedbør-Afstrømings-Model))**

**Input Data:**
- Meteorological data (rainfall and potential evaporation)
- Hydrological data (discharge at the outlet of the catchments for model calibration and validation)
- Model parameters (time constants and threshold values for routing surface storage, rootzone storage and groundwater storage)

**Model Parameter:**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Umax (mm)</td>
<td>Maximum water content in surface storage</td>
<td>mm</td>
</tr>
<tr>
<td>Lmax (mm)</td>
<td>Maximum water content in root zone storage</td>
<td>mm</td>
</tr>
<tr>
<td>CQOF</td>
<td>Overland flow runoff coefficient</td>
<td></td>
</tr>
<tr>
<td>CKIF (h)</td>
<td>Time constant for interflow</td>
<td>h</td>
</tr>
<tr>
<td>TDF</td>
<td>Root zone threshold value for overland flow</td>
<td></td>
</tr>
<tr>
<td>CK1,2 (h)</td>
<td>Time constant for routing overland flow</td>
<td>h</td>
</tr>
<tr>
<td>TIF</td>
<td>Root zone threshold value for interflow</td>
<td></td>
</tr>
<tr>
<td>TG</td>
<td>Root zone threshold value for groundwater recharge</td>
<td></td>
</tr>
<tr>
<td>CKBF (h)</td>
<td>Time constant for routing base flow</td>
<td>h</td>
</tr>
</tbody>
</table>
Case Study of Godavari

HMS Model for Godavari

Carto DEM (30 m)

Soil Texture (NBSS & LUP)

LULC (AWIFS)
Curve Number Grid
Rainfall Pattern for Godavari (June-Sep 2016)

Discharge Hydrograph at Perur
Discharge Hydrograph at Koida

Floodplain Topographic Parameter Extraction

River Geometry
Cross-Sections
Cross-Section Profile
Web-enabled Spatial Flood Early Warning System for the Godavari Basin

Spatial Runoff Pattern
References


Applied Hydrology – Chow, Maidment, Mays, McGraw-Hill


Thank You